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Reg. No. : .....

Name : .....

Fourth Semester B.Sc. Degree Examination, June 2014
First Degree Programme under CBCSS
Complementary Course for Chemistry and Polymer Chemistry
MM 1431.2 : MATHEMATICS IV (Abstract Algebra, Linear Transformations, Coordinate Systems)

Time : 3 Hours Weight : 30

All the first **16** questions are **compulsory**. They carry **4** weightages in **all**.

1. Is Z^+ under addition a group ? Justify.
2. Let G be a group and a in G. Find the smallest subgroup of G that contains a.
3. Define isomerism between two rings R and R'.
4. Find the units in Z_{14}.
5. True or false : The 0-vector can be the part of a basis.
6. Define subspace of a vector space V over F.
7. Define a linear transformation.
8. Define a contraction from $R^2 \rightarrow R^2$.
9. Write the matrix for the reflection through X-axis.
10. Let $T : R^n \rightarrow R^m$ be a one – to-one linear transformation. Write all the solutions of $Tx = 0$.
11. True or False : If A is a 6×5 matrix, then the transformation x to Ax cannot map $R^5 \rightarrow R^6$.
12. Define a shear transformation.
13. Find the spherical coordinates of the cone $z = \sqrt{x^2 + y^2}$.

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14. Find $\int_0^{2\pi}\int_0^{2} r^3 \, dr d\theta$

15. Find $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} l^3 \sin\theta \cos\theta \, dl d\phi d\theta$

16. The Jacobian corresponding to the transformation from Cartesian coordinates to spherical polar coordinate system is given by

Answer **any 8** questions from among the questions **17** to **28**. These questions carry **1** weightage **each**.

17. Show that if G is a finite group of even order, then there is an $a \neq e \in G$ such that $a * a = e$.

18. Prove that every cyclic group is abelian.

19. Compute the product (16) (3) in Z_{32}.

20. Find all the solutions of $x^2 + x - 6 = 0$ in Z_{14}.

21. Give a basis of Q(i) over Q.

22. Prove that $T : R^2 \rightarrow R^2$ defined by $T(x_1, x_2) = (4x_1 - 2x_2, 3|x_2|)$ is not linear.

23. Let T be the linear transformation defined by $T(e_1) = (1, 4)$, $T(e_2) = (-2,9)$ and $T(e_3) = (3, -8)$ where e_1, e_2 and e_3 are the columns of the 3×3 identity matrix. Check whether T is one – to – one or not.

24. Find the values of a such that $\left\{\begin{bmatrix}1\\a\end{bmatrix}, \begin{bmatrix}a\\a+2\end{bmatrix}\right\}$ is linearly dependent.

25. Find the area enclosed by $r = 1 + \cos\theta$.

26. Change $\int_{-1}^{1}\int_{0}^{\sqrt{1+x^2}} dy dx$ to an equivalent polar integral.

27. Evaluate $\int\int_R e^{(x^2+y^2)} dy dx$

28. Find the volume of the solid enclosed by the sphere $x^2 + y^2 + z^2 = 4a^2$ and the planes $z = 0$ and $z = 2a$ using spherical coordinates.

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Answer **any 5** questions from among the questions **29** to **36**. These questions carry **2** weightages **each**.

29. Define * on Q^+, the set of all positive rational numbers, by $a * b = \frac{ab}{4}$. Show that $(Q^+, *)$ is a group.

30. Describe the group D_3 of symmetries of an equilateral triangle.

31. If {u, v, w} is a linearly independent set in R^3. Prove that {u + v, v + w, w + u} is also a linearly independent set.

32. Let $T : R^2 \to R^2$ defined by $T(x_1, x_2) = (x_1 + x_2, 4x_1 + 5x_2)$. Find v such that Tv = (3, 8).

33. Let $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}, c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}, c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$ and consider the bases for R^2 given by $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$. Find the change of coordinates matrix from C to B.

34. Is $\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ diagonalizable ? Justify.

35. Evaluate $\int_0^\infty \int_0^\infty \frac{1}{(1+x^2+y^2)^2} dxdy$.

36. Find the centroid of the region enclosed by the cardioid $r = 1 + \cos\theta$.

Answer **any two** questions from among the questions **37** to **39**. These questions carry **4** weightages **each**.

37. a) If $R_1, R_2, \ldots R_k$ are rings, prove that their direct product is also a ring.

    b) Let R be a ring and a be a fixed element of R. Let $I_a = \{x \in R / ax = 0\}$. Prove that I_a is a subring of R.

38. Diagonalise the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ by finding an invertible matrix P and a diaoganal matrix D such that $A = PDP^{-1}$.

39. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

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